Probability- The Study of Randomness **4.1 and 4.2** + 4.5

Order
4.1
4.2
4.5
4.3
4.4

Intro Vocab:

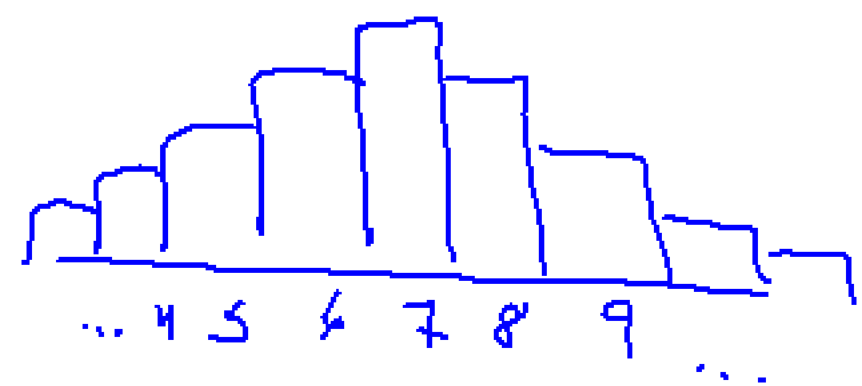
Random (trials)-

individual outcomes are uncertain but in large # trials a regular distribution emerges

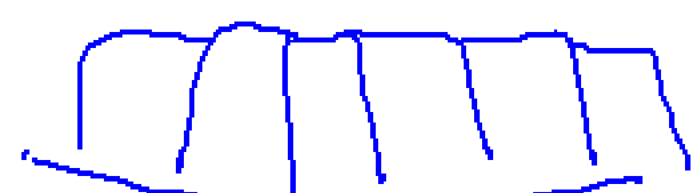
Probability-
(of an event)

prop. of times an event/outcome occurs in large # of trials.

Ex: dice roll x2



dice roll x1



Experimental Probability-

what did happen (in expt.)

Ex: If I toss a coin 30 times, and get 12 heads, what the experimental prob. of getting heads?

$P(H) = 12/30 =$

$P(7) = \frac{}{50}$

Theoretical Probability-

what should happen

Ex: Using the same coin tossing situation above, whats the theoretical prob. of getting heads?

$P(H) = 1/2 = 50\% = 0.5$

$P(7) = 6/36 = 1/6$

as n ↑, the exp. prob becomes closer to theor. prob.

Probability Models- (like distributions)

- lists all possible outcomes
- shows theoretical prob of each outcomes

outcomes →

X	2	3	4	5	...
P(X)	1/36	2/36	3/36	4/36	.....

Sample Space-

the set of all poss. outcomes

$S = \{2, 3, 4, 5, \ldots\ldots 10, 11, 12\}$

$S = \{H, T\}$

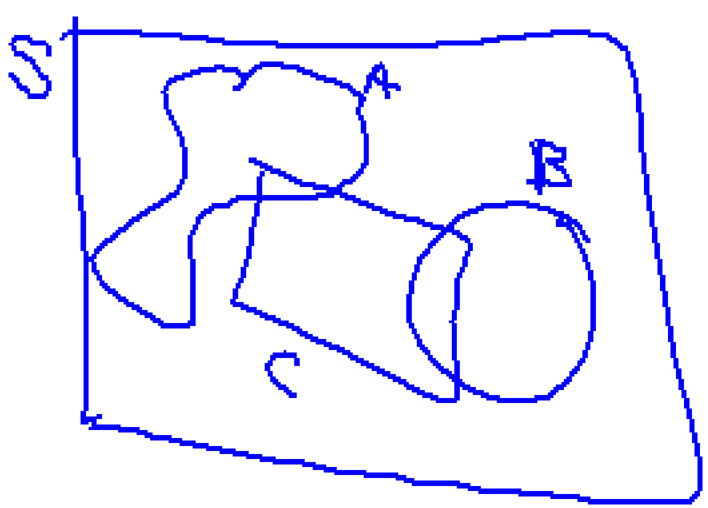
$S = \{$Red, Blue, Green, Yellow$\}$

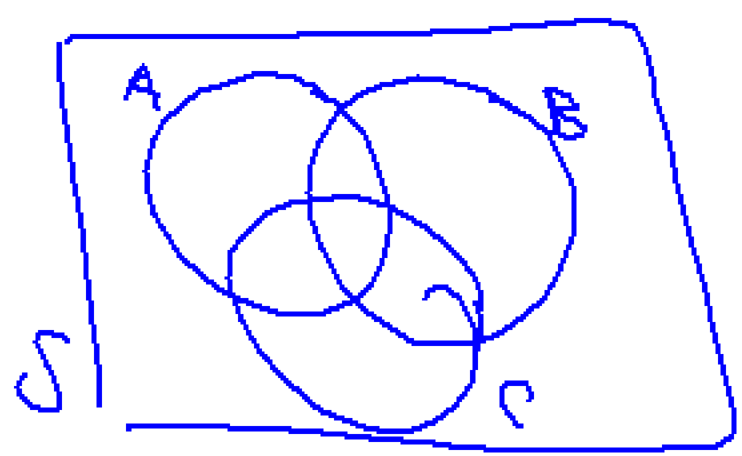
Probability Notation:

- A, B, C, etc. = events
- P(A) = prob. of event A occurring

P(x)

- S = sample space
- When we represent events, we draw them with Venn Diagrams
- Venn Diagrams use shapes to represent events
  w/ a box around representing sample space
  - Examples:

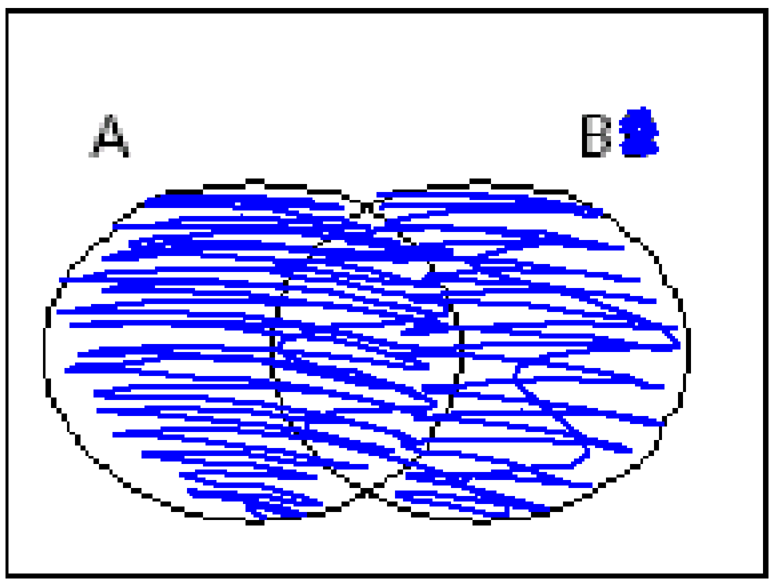




General Set Theory "and"

Union: addition, joining *marriage

- Meaning:
- Symbol: ∪
- Example 1: A ∪ B



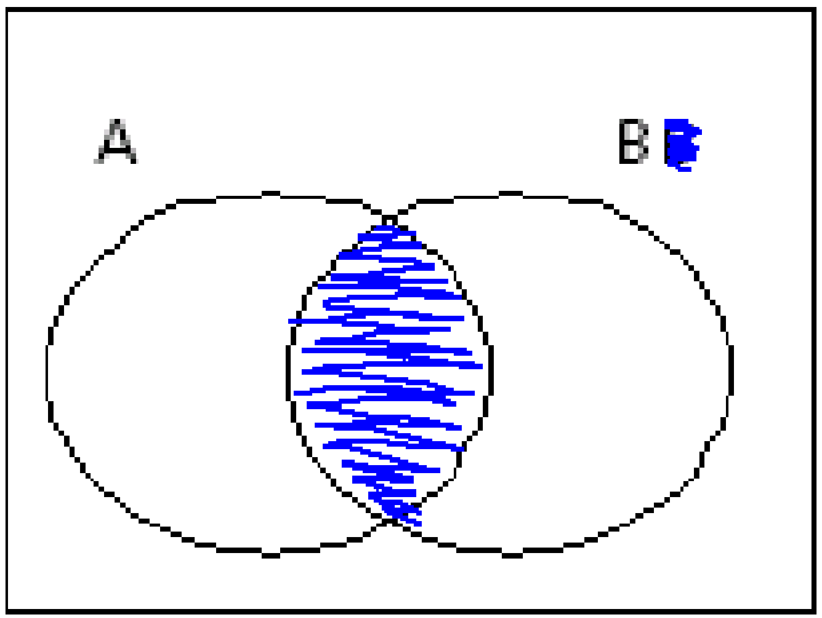
- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8, 10, 12}

*don't double count

Intersection:

- Meaning: overlap, things in common
- Symbol: $\cap$
- Example 1: $A \cap B$



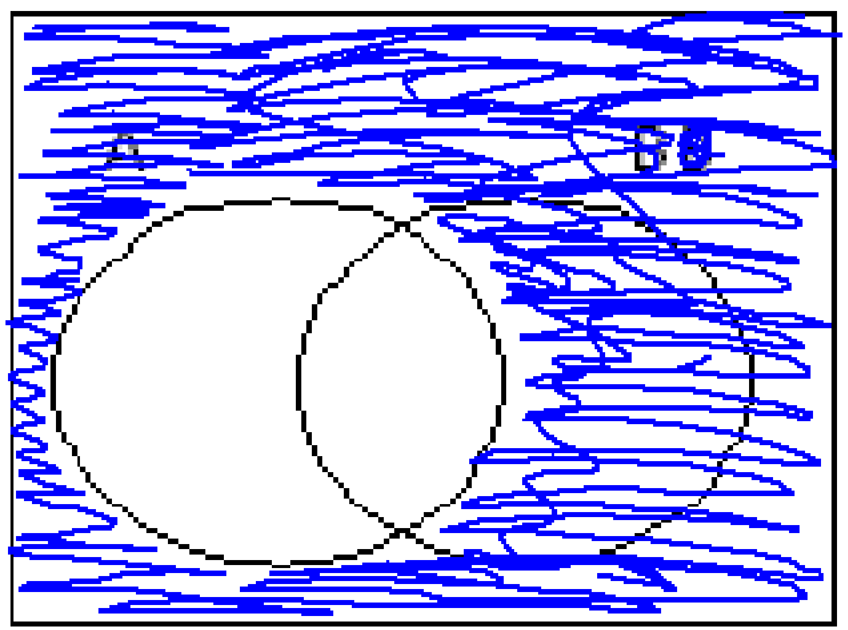
- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

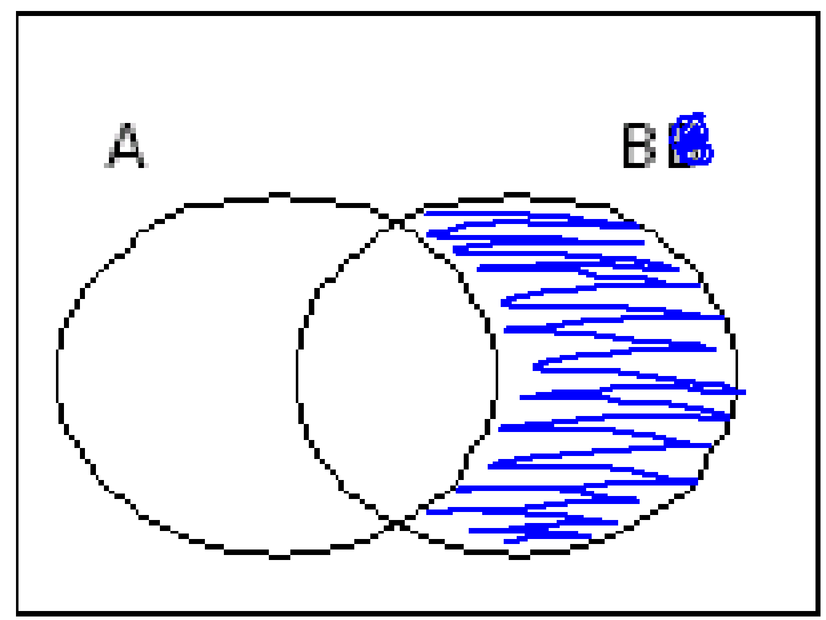
  A∩B = { 2, 4, 6 }

Complement: Of event A

- Meaning:
  - anything not in A, but still in sample space

    "not"
- Symbol: A^c
- Example 1: Shade A^c Shade $A^c \cap B$





- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} = *sample space*

$A^c = \{1, 3, 5, 7, 9, 11, 13, 14, 15\}$

$A \cap B^c$

$A^c \cup B$

$P(A) = \frac{6}{15} =$

$P(A^c) = \frac{9}{15}$

Try the "Set Theory" worksheet

a) $P(E_2) = 1/36$

b) $P(E_2 \cap E_3) = 0$

c) $P(E_4 \cup B) = 9/36 = 1/4$

d) $P(E_3) = 2/36 = 1/18$

e) $P(A) = 6/36 = 1/6$

f) $P(A^c) = 30/36 = 5/6$

g) $P(E_4) = 3/36 = 1/12$

h) $P(B) = 6/36 = 1/6$

i) $P(E_2 \cup B) = 7/36$

Probability Rules

- Let A and B be events
- Let S = sample space
- Let ~~Ac~~ A^c = the complement of event A

List the first 3 probability rules: (page 298)

(1) $0 \leq P(A) \leq \underline{1}$

(2) $P(S) = \underline{1}$ $\quad \sum P(\text{each outcome}) = \underline{1}$

(3) $P(A^c) = 1 - P(A)$

Example 1: If the probability of hitting a homerun is 30%, whats the probability of not hitting a homerun?

* $P(H) =$ 0.30

$P(H^C) =$ 1 - 0.3 = 0.7

Example 2: If there are only 8 different blood types, fill in the chart below:

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	?	0.07	0.1	0.11

0.06

$P(S) = 1$

Example 3:

Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, "UNC's probability of winning is twice Duke's. NC State and UVA each have probability 0.1 of winning, but Duke's probability is three times that. Nobody else has a chance." Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

$P(UNC) = 0.6$

$P(Duke) = 0.3$

$P(NCState) = 0.1$

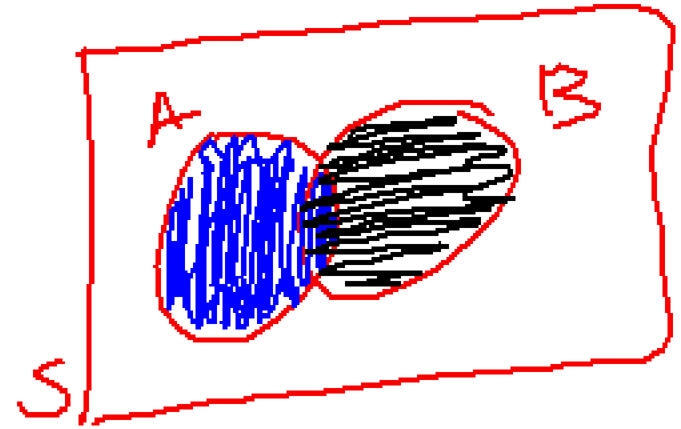
$P(UVA) = 0.1$

$\overline{1.1}$

NO, over 100%

Probability Rules (cont'd)

Unions = addition



General Rule:

* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

*don't want to double count

- Why do we subtract $P(A \cap B)$?

$P(A) = 0.40$

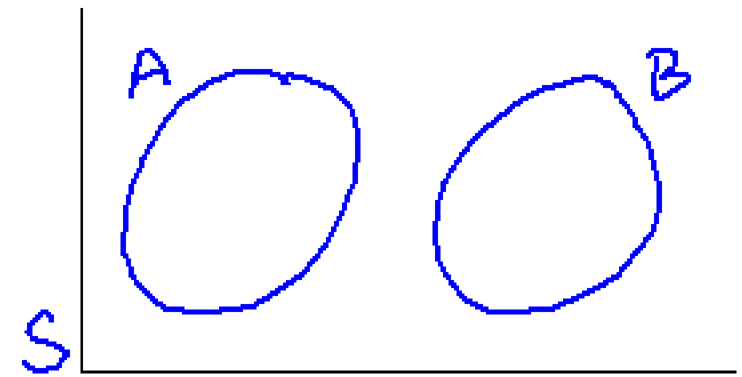
$P(B) = 0.35$

$P(A \cap B) = 0.10$

$P(A \cup B) = 0.4 + 0.35 - 0.10 =$

Special Case:

What if A and B don't overlap? Draw a Venn Diagram that illustrates this below:



So, $P(A \cap B)$ = ? 0

This is called **Disjoint**

Disjoint (or mutually exclusive)= Two events are disjoint if ...

they have no outcomes in common (no overlap)

So our rule for unions for disjoint events then becomes:

- $P(A \cup B) = P(A) + P(B) \cancel{- P(A \cap B)}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Probability Rules (cont'd)

given that

Conditional Probability = P(B|A)= prob. of B given that A happened

Definition:

giving a prob of an event w/ the knowledge of another event.

Formula:

$$P(B|A)= \frac{P(A \cap B)}{P(A)}$$

$$P(G|Q) = \frac{P(G \cap Q)}{P(Q)}$$

Notes:

- $P(A) > 0$
- A = 1st event (after |)
- B = 2nd event

$P(Bl) = 5/10 = 1/2$

$P(R) = 3/10$ *not conditional

$P(R|Bl) = 3/10$

$P(R|Bl) = 3/9$ *conditional

$P(R|Bl) = P(Bl|R) -$

$3/9$ $5/9$

Intersections

General Rule:

- $P(A \cap B) = P(B) \cdot P(A|B)$

$$P(B) \cdot P(A|B) = \frac{P(A \cap B)}{\cancel{P(B)}} \cdot \cancel{P(B)}$$

- ~~Note: A = B =~~

- Also called... multiplication rule

$$P(A \cap B) = P(B) \cdot P(A|B)$$

Special Case:
What if A and B don't affect each other?

Using our example from above, what if we replaced the chips after each pick. What would be the probability of picking a red chip on the first pick? If we picked a blue chip, on the first pick (but replaced it after the pick), what is the probability that we picked a red chip on the second pick? How do these two probabilities (picking red on first and second picks) compare?

So, $P(Red) = 3/10$ and $P(Red|\cancel{Blue}) = 3/10$

*replacement

So in general, $P(B|A) = ?$ $P(B)$

$P(B)$

This is called **Independent**

when 2 events don't affect each other

Independent= Two events are independent if …

the 1st event happening doesn't affect the chance of 2nd event.

Examples of independent events:

Ex: rolling dice

flipping coin

So our rule for intersections for independent events then becomes:

- ~~P(AB) =~~

$P(A \cap B) = P(B) \cdot P(A)$

Flip coin x2

$P(HH) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

roll dice x2

$P(7 \text{ and } 2) = 1/6 \cdot 1/36 =$

Try these:

Probability rules worksheet NAME:______________________

1. If $P(A) = 0.26$ and $P(B) = 0.41$ and $P(A \cap B) = 0.1$, find the following:

   a. $P(A \cup B) =$ $P(A)+P(B)-P(A\cap B) = 0.26+0.41-0.1 = 0.57$

   b. $P(B|A) =$ $\dfrac{P(B\cap A)}{P(A)} = \dfrac{0.1}{0.26} = 0.3846$

   c. Are A and B disjoint events? Why or why not?

   NO $P(A\cap B) \neq 0$

   d. Are A and B independent events? Why or why not?

   NO $P(B|A) \neq P(B)$

2. If P(A) = 0.6 and P(B) = 0.34 and P(B|A) = 0.2, find the following:

a. P(A and B) = $P(A \cap B)$

intersection

b. P(A or B) = $P(A \cup B)$

union

3. Let the sample space, S = {all whole number from 0 through 19}

Let the event A = {2, 4, 6, 8, 10, 12}

Let the event B = {3, 6, 9, 12, 15, 18}

Let the event C = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

Let the event D = {1, 4, 7, 8, 10, 14, 16, 18}

Find the following:

a. A   B =

b. P(A   B) =

c. D^c =

d. P(C   B) =

e. P(A U B) =

f. P(C   D) =

g. P (C^c) =

h. C U A =